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Covariant Conservation Laws from the Palatini Formalism

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Covariant conservation laws in the Palatini formalism are derived. The result indicates that the gravitational part of conserved charges in general relativity should be calculated from a combination of Komar's strongly conserved current and the Einstein tensor. This implies that the set of complete diffeomorphism charges of a gravitating system consisting of scalar matter is described by Komar's vector density, and that the identification of gravitational energy and momentum reduces to two choices: a choice of relative weights of the contributions resulting from Komar's current and from the Einstein tensor, and a choice of preferred vector fields in space-time. A proposal is made which yields energy and momentum as scalars under diffeomorphisms and as a Lorentz vector in tangent space. Furthermore, the result can be used to identify covariant conservation laws holding separately for the matter contributions to diffeomorphism charges.

1. INTRODUCTION

The ambiguities in the definition of energy and momentum of the gravitational field provide a still incompletely understood aspect of general relativity. A classical and still useful survey of the development until 1962 is provided in Trautman (1962). Useful accounts of the subject, each from a different point of view and emphasizing different results, can also be found in Schmutzer (1968), Møller (1972), Weinberg (1972), Misner *et al.* (1973), Wald (1984), Carmeli *et al.* (1990), and De Felice and Clarke (1990). However, none of the solutions of this long-standing problem is considered satisfactory, and there is still active research on the identification of conserved charges in general relativity, yielding interesting new ideas and developments. Recent contributions can be found in articles by Murphy (1990), who investigates a conserved complex from the Palatini formalism, and by Nissani and Leibowitz (1991), who split the energy-momentum tensor $T_{\mu\nu}$

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into contributions from matter and from the gravitational field. Conserved charges from $SL(2, \mathbb{C})$ gauge theory are discussed in Carmeli *et al.* (1990). Of interest also is the beautiful idea of Nissani and Leibowitz to identify nonrotating frames, where conservation of the energy-momentum tensor globally takes the form $\partial_\mu T^{\mu\nu}=0$; see Nissani and Leibowitz (1988, 1989, 1991) and Carmeli *et al.* (1990).

Here I am interested in a discussion of the conservation laws resulting from the invariance of general relativity under reparametrizations $x \to \xi(x)$, which are supposed to represent diffeomorphisms (of appropriate order) of the space-time manifold.

According to standard conventions, a conservation law related to a current j is denoted as strong if $\partial_\mu j^\mu=0$ holds identically, and as weak if $\partial_\mu j^\mu$ is proportional to the equations of motion. Furthermore, a conservation law $\partial_\mu j^\mu=0$ is denoted as covariant if the current j is a vector density. In this case, and if the space like components of j vanish sufficiently fast, then j defines a conserved charge

$$Q=-\int_{\mathscr{S}} * \frac{j}{\sqrt{-g}}$$

which is invariant. Here $\mathscr{S}$ is a spacelike hypersurface, and

$$(*v)_{\nu\rho\sigma} \equiv v^\mu \varepsilon_{\mu\nu\rho\sigma}$$

Extending earlier work by Bergmann (1958) and Møller (1958) on conservation laws in general relativity, Komar (1959) proposed the strongly conserved current

$$K^\mu = 2\sqrt{-g}(\nabla_\nu \nabla^\mu \varepsilon^\nu - \Delta \varepsilon^\mu) = 2\partial_\nu[\sqrt{-g}(\nabla^\mu \varepsilon^\nu - \nabla^\nu \varepsilon^\mu)]$$

to be considered as a vector density of charges related to the reparametrizations

$$x^\mu \to \xi^\mu(x) = x^\mu + \varepsilon^\mu(x)$$

This work was based (in an indirect manner) on the Lagrangian

$$\mathscr{L}(g, \partial g) = \frac{1}{2\chi}\sqrt{-g}(\Gamma^\mu{}_{\nu\lambda}\Gamma^\nu{}_{\mu\sigma} - \Gamma^\mu{}_{\nu\mu}\Gamma^\nu{}_{\lambda\sigma})g^{\lambda\sigma}$$

However, soon thereafter, Fletcher (1960) derived Komar's expression for pure gravity directly from the Einstein–Hilbert Lagrangian. The result in the case $\partial_\mu \varepsilon^\nu=0$ was also obtained already by Mizkjewitsch (1958).

In the sequel I will discuss diffeomorphism charges and covariant conservation laws derived from the Palatini formalism, thus extending the recent work of Murphy (1990), who derived the conservation laws in the case $\partial_\mu \varepsilon^\nu = 0$. The motivation for this work is twofold: On one hand, the identification of covariant conservation laws from the Palatini formalism includes the covariant conservation laws of general relativity, and thus may help to clarify the issue of energy and momentum of the gravitational field. On the other hand, there exist similarities between the Palatini formalism in general relativity and the Polyakov action in string theory, where the conservation laws derived from the Polyakov action play a crucial role in the quantization of the theory [see Dick (1991) for a discussion of superstring theory on arbitrary genus and in a covariantized primary field formalism].

The development leading to Komar's proposal and the direct construction of conserved charges from the Einstein–Hilbert Lagrangian is briefly reviewed in Sections 2 and 3, while in Section 4 the covariant conservation laws from the Palatini formalism are introduced. Section 4 contains also a proposal on the identification of gravitational energy and momentum vector densities among the conserved currents, which yields an energy-momentum Lorentz vector in tangent space.

The conventions employed in this paper follow Misner *et al.* (1973). Furthermore, torsion is always excluded, and covariant derivatives which explicitly preserve the metric under parallel translation are denoted by ∇, i.e., $\nabla g = 0$, while otherwise they are denoted by D.

2. CONSERVATION LAWS ACCORDING TO EINSTEIN, BERGMANN, MØLLER, AND KOMAR

A usual canonical derivation of conserved charges of a gravitational field employs the familiar Noether theorem for a reparametrization $x^\mu \to \xi^\mu(x) = x^\mu + \varepsilon^\mu(x)$. If the first-order change of the fields is denoted by $\varphi(x) \to \Phi(\xi) = \varphi(x) + \delta\varphi(x)$, then the Noether theorem is conveniently expressed in the following equation for the first-order change of the action $S = \int d^4x\, \mathscr{L}(\varphi, \partial\varphi)$:

$$\delta S = \int d^4x \left[\partial_\mu \left(\varepsilon^\mu \mathscr{L} + (\delta\varphi_I - \varepsilon^\nu \partial_\nu \varphi_I) \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi_I)} \right) \right.$$
$$\left. + (\delta\varphi_I - \varepsilon^\nu \partial_\nu \varphi_I) \left(\frac{\partial \mathscr{L}}{\partial \varphi_I} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi_I)} \right) \right] \tag{1}$$

where φ_I labels the different fields appearing in $\mathscr{L}$, and all repeated indices are summed over. Conserved charges arising via the Noether theorem from a reparametrization symmetry of the equations of motion will be denoted as diffeomorphism charges in the sequel.

In the realm of gravitation theory the Noether theorem has been particularly applied to the Lagrangian

$$\mathscr{L}(g, \partial g)=\frac{1}{2\chi}\sqrt{-g}\,(\Gamma^{\mu}{}_{\nu\lambda}\Gamma^{\nu}{}_{\mu\sigma}-\Gamma^{\mu}{}_{\nu\mu}\Gamma^{\nu}{}_{\lambda\sigma})g^{\lambda\sigma} \tag{2}$$

which differs by a complete divergence from the Einstein–Hilbert Lagrangian. Clearly, $\mathscr{L}(g, \partial g)$ is not a scalar density under diffeomorphisms, and, hence, δS has to be evaluated carefully, before conservation laws can be derived from equation (1).

It is well known that the early results of Einstein on conserved charges in general relativity can be derived by exploiting the fact that $\delta(\mathscr{L}(g, \partial g)/\sqrt{-g})=0$, hence $\delta S=0$, under linear reparametrizations. Consideration of the case $\varepsilon^{\mu}=\text{const}$ yields particularly the weakly conserved Einstein complex

$$\begin{aligned}\sqrt{-g}\,t_{\alpha}{}^{\beta}&=g_{\alpha}{}^{\beta}\mathscr{L}-(\partial_{\alpha}g^{\mu\nu})\frac{\partial\mathscr{L}}{\partial(\partial_{\beta}g^{\mu\nu})}\\&=g_{\alpha}{}^{\beta}\mathscr{L}-\frac{1}{2\chi}[\Gamma^{\mu}{}_{\rho\mu}\partial_{\alpha}(\sqrt{-g}\,g^{\rho\beta})-\Gamma^{\beta}{}_{\rho\mu}\partial_{\alpha}(\sqrt{-g}\,g^{\rho\mu})]\end{aligned} \tag{3}$$

Bergman (1958) pointed out that it is possible to construct conservation laws from equations (1) and (2) for arbitrary reparametrizations due to the fact that the first-order change of δS is given by a complete divergence:

$$\delta S=\int d^4x\,\sqrt{-g}\;\delta\frac{\mathscr{L}(g, \partial g)}{\sqrt{-g}}=\int d^4x\;\partial_{\mu}k^{\mu} \tag{4}$$

A direct calculation from equation (2) yields

$$k^{\mu}=\frac{1}{2\chi}(g_{\rho}{}^{\mu}g_{\lambda}{}^{\sigma}-g_{\rho}{}^{\sigma}g_{\lambda}{}^{\mu})(\partial_{\nu}\varepsilon^{\lambda})\partial_{\sigma}(\sqrt{-g}\,g^{\rho\nu}) \tag{5}$$

Insertion into (1) implies a weak conservation law related to the current

$$C^{\mu}=\varepsilon^{\mu}\mathscr{L}+2(\nabla^{\nu}\varepsilon^{\rho})\frac{\partial\mathscr{L}}{\partial(\partial_{\mu}g^{\nu\rho})}-k^{\mu}=\sqrt{-g}\;\varepsilon^{\alpha}t_{\alpha}{}^{\mu}+\frac{1}{2\chi}U_{\alpha}{}^{\mu\nu}\partial_{\nu}\varepsilon^{\alpha} \tag{6}$$

with the superpotential

$$U_{\alpha}{}^{\mu\nu}=\frac{1}{\sqrt{-g}}g_{\alpha\beta}\partial_{\rho}[g(g^{\beta\nu}g^{\rho\mu}-g^{\beta\mu}g^{\rho\nu})] \tag{7}$$

Furthermore, Bergmann employed the equations of motion to write the conserved current (6) in compact form as

$$C^{\mu} = \frac{1}{2\chi}\,\partial_{\nu}(\varepsilon^{\alpha} U_{\alpha}{}^{\mu\nu}) \tag{8}$$

However, C^{μ} does not represent a vector density and, hence, the conserved charges calculated from (8) have undesirable transformation properties. Now, the weak equivalence $2\chi\sqrt{-g}\,t_{\alpha}{}^{\mu} = \partial_{\nu} U_{\alpha}{}^{\mu\nu}$ was also exploited by Møller (1958) to define an improved complex via

$$\sqrt{-g}\,\tau_{\alpha}{}^{\mu} = \frac{1}{2\chi}\,\partial_{\nu} V_{\alpha}{}^{\mu\nu} \tag{9}$$

where he defined the superpotential

$$V_{\alpha}{}^{\mu\nu} = 2U_{\alpha}{}^{\mu\nu} - \delta_{\alpha}{}^{\mu} U_{\rho}{}^{\rho\nu} + \delta_{\alpha}{}^{\nu} U_{\rho}{}^{\rho\mu} \tag{10}$$

Møller demonstrated that this definition yields charges $\{Q^{0}, \mathbf{Q}\}$ with the property that Q^{0} is invariant under time-independent reparametrizations, while $\mathbf{Q}$ transforms like a 3-vector. The transformation properties of $\{Q^{0}, \mathbf{Q}\}$ were further discussed by Kovacs (1985).

The results (8) and (9) of Bergmann and Møller motivated Komar (1959) to investigate the currents $\partial_{\nu}(\varepsilon^{\alpha} V_{\alpha}{}^{\mu\nu})$, and he demonstrated that addition of a further curl field yields the vector density

$$K^{\mu} = 2\sqrt{-g}\,(\nabla_{\nu}\nabla^{\mu}\varepsilon^{\nu} - \Delta\varepsilon^{\mu}) \tag{11}$$

which he suggested to be considered as the conserved currents related to infinitesimal reparametrizations $x^{\mu} \to \xi^{\mu}(x) = x^{\mu} + \varepsilon^{\mu}(x)$.

Exploiting the identity

$$\varepsilon_{\mu\alpha\beta\gamma}(\nabla_{\nu}\nabla^{\mu}\varepsilon^{\nu} - \Delta\varepsilon^{\mu}) = 3\nabla_{[\alpha}\varepsilon_{\beta\gamma]\mu\nu}\nabla^{\mu}\varepsilon^{\nu}$$

demonstrates that Komar's invariant charges can be calculated as integrals over boundaries:

$$Q = -\int_{\partial\mathscr{S}} dx^{\mu} \wedge dx^{\nu}\ \varepsilon_{\mu\nu\rho\sigma}\nabla^{\rho}\varepsilon^{\sigma} = -2\int_{\partial\mathscr{S}} *\,d\varepsilon$$

The synopsis of the development leading from the Einstein complex to Komar's covariant conservation laws reveals that all the conserved currents can be written as strongly conserved currents plus terms of the form $\varepsilon^{\mu}(R/2) - R^{\mu}{}_{\nu}\varepsilon^{\nu}$. This holds also for the Landau–Lifshitz complex (see, e.g., Møller, 1972) and for the complex of Brill and Deser (see, e.g., Weinberg, 1972). The property of pure gravitational currents to differ from the Einstein tensor by strongly conserved currents is brought most clearly to the light by a direct construction of diffeomorphism charges from the Einstein–Hilbert Lagrangian.

3. DIFFEOMORPHISM CHARGES FROM THE EINSTEIN–HILBERT LAGRANGIAN

To derive the conserved charges related to the reparametrization invariance of the Einstein–Hilbert Lagrangian, it is useful first to generalize equation (1) to Lagrangians including second-order derivatives (Mizkjewitsch, 1958; Fletcher, 1960; Schmutzer, 1961):

$$\delta S=\int d^4x\Bigg[\partial_\mu\left(\varepsilon^\mu\mathscr{L}+(\delta\varphi_I-\varepsilon^\nu\partial_\nu\varphi_I)\frac{\partial\mathscr{L}}{\partial(\partial_\mu\varphi_I)}-(\delta\varphi_I-\varepsilon^\nu\partial_\nu\varphi_I)\bar{\partial}_\rho\frac{\partial\mathscr{L}}{\partial(\partial_\rho\partial_\mu\varphi_\iota)}\right)$$
$$+(\delta\varphi_I-\varepsilon^\nu\partial_\nu\varphi_I)\left(\frac{\partial\mathscr{L}}{\partial\varphi_I}-\partial_\mu\frac{\partial\mathscr{L}}{\partial(\partial_\mu\varphi_I)}+\partial_\rho\partial_\mu\frac{\partial\mathscr{L}}{\partial(\partial_\rho\partial_\mu\varphi_I)}\right)\Bigg] \tag{12}$$

where $f\bar{\partial}g\equiv f\partial g-g\partial f$.

Application of this result to the Einstein–Hilbert Lagrangian

$$\mathscr{L}(g,\partial g,\partial\partial g)=\frac{1}{2\chi}\sqrt{-g}\,R \tag{13}$$

is lengthy and tedious, but straightforward, so I will only remind the reader that the curvature scalar can be written as

$$R=g^{\mu\nu}g_{\alpha\beta}\partial_\mu\partial_\nu g^{\alpha\beta}-\partial_\mu\partial_\nu g^{\mu\nu}+\partial_\mu g^{\mu\nu}\cdot g_{\alpha\beta}\partial_\nu g^{\alpha\beta}+\tfrac{1}{2}g_{\alpha\beta}\partial_\mu g^{\alpha\nu}\cdot\partial_\nu g^{\beta\mu}$$
$$-\tfrac{5}{4}g^{\mu\nu}g_{\alpha\beta}g_{\gamma\delta}\partial_\mu g^{\alpha\gamma}\cdot\partial_\nu g^{\beta\delta}-\tfrac{1}{4}g^{\mu\nu}g_{\alpha\beta}\partial_\mu g^{\alpha\beta}\cdot g_{\gamma\delta}\partial_\nu g^{\gamma\delta} \tag{14}$$

The weakly conserved current resulting from equation (12) is then

$$j^\mu=\frac{1}{2\chi}\sqrt{-g}\,[2\nabla^\mu\nabla_\nu\varepsilon^\nu-\nabla_\nu\nabla^\mu\varepsilon^\nu-\Delta\varepsilon^\mu+\varepsilon^\mu R] \tag{15}$$

Application of the relation

$$[\nabla_\nu,\nabla^\mu]\varepsilon^\nu=R^\mu{}_\nu\varepsilon^\nu \tag{16}$$

yields

$$j^\mu=\frac{1}{2\chi}\sqrt{-g}\,[\nabla_\nu\nabla^\mu\varepsilon^\nu-\Delta\varepsilon^\mu-2(R^\mu{}_\nu-\tfrac{1}{2}g^\mu{}_\nu R)\varepsilon^\nu] \tag{17}$$

implying that the gravitational part of any diffeomorphism charge consists of a trivially conserved part and a term proportional to the Einstein tensor density. Note that the Einstein tensor cannot be omitted in (17), because if a matter Lagrangian is added to the Einstein–Hilbert Lagrangian, then the Einstein tensor will yield a nonvanishing contribution to the *gravitational* part of the diffeomorphism charges, which is exactly what we are seeking.

On the other hand, it is tempting to conclude from equation (17) that the gravitational part of diffeomorphism charges is simply given by

$$\begin{aligned} j^\mu &= -\frac{1}{\chi}\sqrt{-g}\,(R^\mu{}_\nu - \tfrac{1}{2}g^\mu{}_\nu R)\varepsilon^\nu = -\frac{1}{\chi}\sqrt{-g}\;G^\mu{}_\nu\varepsilon^\nu \\ &= -2g^{\mu\rho}\,\frac{\delta\mathscr{L}(g,\partial g,\partial\partial g)}{\delta g^{\rho\nu}}\,\varepsilon^\nu \end{aligned} \tag{18}$$

Motivated by the similarity of this expression to the energy-momentum tensor of matter

$$\sqrt{-g}\;T^\mu{}_\nu\varepsilon^\nu = -2g^{\mu\rho}\,\frac{\delta\mathscr{L}(\varphi,\partial\varphi)}{\delta g^{\rho\nu}}\,\varepsilon^\nu \tag{19}$$

the interpretation of $-G_{\mu\nu}/\chi = (g_{\mu\nu}R - 2R_{\mu\nu})/(2\chi)$ as the energy-momentum tensor of the gravitational field was suggested long ago by Levi-Civita and Lorentz.

Hence, from this point of view the vector density which expresses the invariance of a scalar system under reparametrizations $x \to \xi(x) = x + \varepsilon(x)$ is given by

$$\mathscr{j}_\mu = \sqrt{-g}\left(T_{\mu\nu} - \frac{1}{\chi}G_{\mu\nu}\right)\varepsilon^\nu \tag{20}$$

The Einstein equation then would state that a space-time manifold is in equilibrium if the globally vanishing diffeomorphism charges are distributed uniformly. However, at this stage it is too early for a judicious choice on how to weigh the strongly conserved current of Komar versus the Einstein tensor, and in Section 4 we will find some evidence that the gravitational part of diffeomorphism charges has to be calculated from the full current displayed in equation (17).

4. COVARIANT CONSERVATION LAWS IN THE PALATINI FORMALISM

The metric appears in the Palatini action

$$\mathscr{L}(g,\Gamma,\partial\Gamma) = \frac{1}{2\chi}\sqrt{-g}\,g^{\mu\nu}(\partial_\rho\Gamma^\rho{}_{\mu\nu} - \partial_\nu\Gamma^\rho{}_{\mu\rho} + \Gamma^\rho{}_{\lambda\rho}\Gamma^\lambda{}_{\mu\nu} - \Gamma^\rho{}_{\lambda\nu}\Gamma^\lambda{}_{\mu\rho}) \tag{21}$$

like an auxiliary field ensuring a vanishing "energy-momentum" tensor density

$$-\frac{1}{\chi}\sqrt{-g}\,G_{\mu\nu}=-2\,\frac{\delta\mathscr{L}(g,\Gamma,\partial\Gamma)}{\delta g^{\mu\nu}}$$

However, the conservation laws related to the invariance of the Palatini action under $x\rightarrow\xi(x)=x+\varepsilon(x)$ assume a more complicated form: The conserved current

$$J^{\mu}=\varepsilon^{\mu}\mathscr{L}(g,\Gamma,\partial\Gamma)+(\delta\Gamma^{\alpha}{}_{\beta\gamma}-\varepsilon^{\nu}\partial_{\nu}\Gamma^{\alpha}{}_{\beta\gamma})\,\frac{\partial\mathscr{L}(g,\Gamma,\partial\Gamma)}{\partial(\partial_{\mu}\Gamma^{\alpha}{}_{\beta\gamma})}\tag{22}$$

can be written in an explicitly covariant way by use of

$$\delta\Gamma^{\alpha}{}_{\beta\gamma}-\varepsilon^{\nu}\partial_{\nu}\Gamma^{\alpha}{}_{\beta\gamma}=-D_{\gamma}D_{\beta}\varepsilon^{\alpha}+R^{\alpha}{}_{\beta\gamma\nu}\varepsilon^{\nu}\tag{23}$$

Thus we find the following vector density of conserved currents:

$$J^{\mu}=\frac{1}{2\chi}\sqrt{-g}\,[\varepsilon^{\mu}R+g^{\rho\sigma}(R^{\mu}{}_{\rho\sigma\nu}\varepsilon^{\nu}-D_{\rho}D_{\sigma}\varepsilon^{\mu})+g^{\mu\rho}(D_{\nu}D_{\rho}\varepsilon^{\nu}-R_{\rho\nu}\varepsilon^{\nu})]\tag{24}$$

Insertion of the metric-preserving connection into (24) reproduces the conserved current displayed in equation (17). However, there is an important difference between the currents displayed in equations (17) and (24): The generalization of Komar's vector density appearing in equation (24) yields nontrivial contributions to the conservation of diffeomorphism charges, because its divergence no longer vanishes off-shell, and hence it should no longer be omitted by a triviality argument. Although this result cannot be considered as a proof, it provides evidence in favor of the conjecture that the gravitational current of diffeomorphism charges should be described by the already introduced combination of Komar's current and the Einstein tensor:

$$j_{\mu}=\frac{1}{2\chi}\sqrt{-g}\,[\nabla_{\nu}\nabla_{\mu}\varepsilon^{\nu}-\Delta\varepsilon_{\mu}-2G_{\mu\nu}\varepsilon^{\nu}]\tag{25}$$

From equation (12) we also note that the matter contribution to currents of diffeomorphism charges generically will be of the form

$$\Theta^{\mu}=\sqrt{-g}\,T^{\mu}{}_{\nu}\varepsilon^{\nu}+\delta\psi_{I}\,\frac{\partial\mathscr{L}}{\partial(\partial_{\mu}\psi_{I})}\tag{26}$$

For a gravitating system consisting of scalar matter this implies that Komar's strongly conserved current should be considered as the *complete* current of diffeomorphism charges. In passing I would also like to remark that we may derive from (25) nontrivial conservation laws which hold separately for Θ^{μ}.

This usually requires the insertion of Killing vectors in the evaluation of the conserved currents, because only the dynamical evolution of matter in a prescribed space-time is taken into account. However, the proposal to employ Killing vectors is very restrictive, as, e.g., in Friedmann models there exist (apart from the empty flat case) only six spacelike Killing vectors corresponding to spacelike rotations and generalized translations. Here the requirement is much weaker, because we know by construction that the equation $\partial_\mu(j^\mu+\Theta^\mu)=0$ holds weakly, and therefore

$$G^{\mu\nu}\nabla_\mu\varepsilon_\nu=0 \tag{27}$$

is both sufficient and necessary to ensure separate conservation of gravitational and matter contributions to diffeomorphism charges. For a Robertson–Walker metric, e.g.,

$$ds^2=-dt^2+a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\vartheta^2+\sin^2\vartheta\ d\varphi^2)\right]$$

equation (27) takes the following form:

$$\begin{aligned} a^4G^{\mu\nu}\nabla_\mu\varepsilon_\nu=3a\ \partial_t[(\dot a^2+k)a\varepsilon_t]-\frac{1}{r^2}(2a\ddot a+\dot a^2+k)\\ \times\Bigg[\sqrt{1-kr^2}\,\partial_r(r^2\sqrt{1-kr^2}\ \varepsilon_r)\\ +\frac{1}{\sin\vartheta}\ \partial_\vartheta(\sin\vartheta\ \ \varepsilon_\vartheta)+\left(\frac{1}{\sin\vartheta}\right)^2\partial_\varphi\varepsilon_\varphi\Bigg]=0 \end{aligned}$$

which simply fixes ε_t for any prescribed set of spacelike components.

However, the identification of covariant conservation laws in general relativity is only one facet of the problem: There remains the task to select energy and momentum vector densities among the conserved currents. The definition of energy and momentum in special relativity and the principle of equivalence suggest the following choice: Set up a continuous distribution of inertial frames. Then insert those anholonomic vector fields ε which correspond to orthonormal bases in the intertial frames to calculate gravitational energy and momentum densities from (25). If $e_\mu{}^a$ is a tetrad, then this amounts to the following vector density of a gravitational energy-momentum vector:

$$P^\mu_a=\frac{1}{2\chi}\sqrt{-g}\,[\nabla_\nu\nabla^\mu e^\nu{}_a-\Delta e^\mu{}_a-2G^\mu{}_\nu e^\nu{}_a] \tag{28}$$

yielding a gravitational energy-momentum vector

$$\mathcal{P}_a = -\int_{\mathcal{S}} * \frac{1}{\sqrt{-g}} P_a \tag{29}$$

for a spacelike hypersurface $\mathcal{S}$. Note that $\mathcal{P}$ is a Lorentz vector of diffeomorphism scalars, thus providing a suggestive answer to the old, but still controversially discussed problem of what kind of mathematical entity gravitational energy-momentum will correspond to.

In conclusion, we observe that all the canonical derivations of diffeomorphism charges in general relativity yield the result (25), if noncovariant superpotentials are excluded. Nevertheless, we still have the freedom to multiply the strongly conserved part by a weight factor, and we have to select those vector fields ε in space-time which yield the gravitational energy and momentum when inserted into the gravitational energy-momentum operator ($\alpha = 1$ above):

$$\sqrt{-g}\, \boldsymbol{\tau} = \frac{\alpha}{2\chi} \sqrt{-g}\, [\tfrac{1}{2}(\{\nabla, \nabla\} + \mathbf{R}) - \Delta \mathbf{g}] - \frac{1}{\chi}\sqrt{-g}\, \mathbf{G}$$

Though the choice proposed in equation (28) seems most natural to me, to resolve the remaining ambiguities requires further investigations both in "experimental" directions, namely on the significance of $\boldsymbol{\tau}$ in exactly solved models, and in theoretical directions, particularly on possible current algebras based on $\boldsymbol{\tau}$.

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